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THE

SOLUTION

OF THE

Quadrature of the Circle.

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BRITISH
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TO THE

READER.

THAT the Circle muſt be regular and exact in its Proportions, is, I think, paſt a Doubt, as it is the principal Figure in Nature; and if I may be allowed the Expreſſion, ſeems to be the Mould in which the whole Creation is formed. This very Reflection was the Cauſe of this Production.

I was in hopes to have preſented this Solution to the Public in a different Manner; and I once did not doubt, but that I could almoſt any where have procured Friends, to have aſſiſted me in forwarding a Work of this Nature, but I was greatly miſtaken.

For, though it is a Problem, if truly ſolved, which will be of much Conſequence to the SCIENCES, and which has for Ages been eagerly and earneſtly ſought after: Yet it is alſo a Problem which has been canvaſſed by ſuch able Heads, and which has been ſo long conſidered to be paſt finding out, that no Wonder it ſhould be thought, not only Preſumption, but the Height of Abſurdity, for almoſt any one even to attempt it, much more for a Perſon entirely unknown and unheard of in the learned World, therefore not worth Notice. But the Race is not always to the ſwift, neither is the Battle always to the ſtrong; neither have all Men the ſame Mode of Thinking.

If thoſe able Men, whoſe mental Powers were ſo amazingly great, if they could but have deſcended into the ſame Train of Thinking; what has been great Labour and Study to me, would have been nothing with them.

But it does not follow becauſe they failed, that other Methods may not be diſcovered which may anſwer better. Theirs has now been tried long enough, and we ſee has failed, therefore moſt likely is ſomehow wrong, though ſublime indeed!

Mine though ſimple, will, I hope, be found to come at Truth, which is what theirs never has arrived at. If we would but conſider the Proportions, in, and about the Circle itſelf, inſtead of attributing to it, Fancies of our own, and perplexing our Heads with Infiniteſimals, which few, if any, can underſtand, and which we ſee have not anſwered the Purpoſe, we ſhould not find this Problem ſo difficult to be come at, as is uſually imagined.

I wiſh that my Reader could diveſt himſelf of all Prejudices which he may have imbibed for thoſe great Authorities, and that he would think a little for himſelf, and not conclude, becauſe thoſe great Men miſſed it, that it is impoſſible to be done.

All I deſire is, that he will lay aſide ſuch Prejudices, and that he will only examine with Candour what I here offer to him, and then let him determine upon it.

I do believe it has been owing to ſome ſuch Prejudices that I have ſo far met with little elſe but Diſcouragements and Neglects.

Notwithſtanding which, as I do verily believe my Solution to be true, I am determined if it is ſo, that it ſhall not be loſt to the Public.

I have laboured hard in ſearching after Truth, and Truth has been my only Aim; whether or no I have found her, I now ſubmit entirely to the Public, and I hope they will not be ſevere upon me.

As I ſhould be ſorry that any one paid Money for any Errors of mine; I intend, before it is advertiſed for Sale, to diſtribute Copies to different Parts of the Kingdom, by which means it will ſoon be known, whether it is true or falſe; and I take this Method that I may alſo be ſatisfied myſelf, as I find that I cannot obtain Satisfaction by applying to Individuals; if it is an Error, there is an End of it, and no one will have been cheated or injured by it; for the Labour, and the Expences attending it, will have been all my own.

But if, on the contrary, it ſhould be found true, which I have great Hopes of; Copies, if wanted, may be had as directed in the Title Page.

THE

INTRODUCTION.

A Right Line can touch a Circle, but in one Point only, and a Point is defined to have no Parts or Magnitude; therefore, if we take any Meaſure for ever ſo ſmall a Portion of the Circumference, we certainly muſt take more than a Point, and more than one Point cannot be a right Meaſure; for a right Line drawn to any two Points in the Circumference, will fall within the Circle, and, conſequently, muſt leſſen its Capacity. And ſo we find it, for the Circle always yet has exceeded in Practice, what it has been ſuppoſed in Theory. Since therefore, to this time, it has been found inſufficient to come at the true Contents of the Circle, by means of the Circumference; and, ever will be ſo, except we can come at its true Value, which ſeems, from what has been ſaid above, to be impoſſible, by any Diviſions of it whatſoever; for, if we reduce the Circumference down to one Point, we cannot

EUCLID.
Coroll. 2 of 3.
Def. 1 of 1.
2 of 3.

cannot ſay what that is; and, if we take more than a Point for a Meaſure, we ſee the Conſequence, we muſt leſſen its Capacity. Therefore, if ever it is found out, it muſt be by ſome other Means. Whether I have been ſo fortunate as to have diſcovered the right Mean, will be ſeen hereafter; but, what I have here to offer, (ſo far as I have been able to apply it) appears to me, entirely to correſpond both in Theory and Practice.

THE

SOLUTION.

IN and about this Circle ſeveral Lines are drawn, the two principal ones, are the two Diameters, *G A*, Fig. I. *P I*, the Square of either of theſe Lines, is equal to that Square which circumſcribes the Circle.

The Line *G P*, is a Chord to the fourth Part of the Circumference, and the Square of this Line, is equal to the inſcribed Square within the Circle, and is alſo equal to half the Square of the Diameter *G A*, as appears upon the View of it, therefore needs no other Proof.

The Secant Line *G S*, cuts the Circle in the Point *B*.

And laſtly, The Line *B C*, is drawn parallel to the 31 of 1. Tangent Line *S A*, through the Point *B*.

Now, I ſay, that the Square of the Line *G B*, which the Circle cuts off from the Secant Line *G S*, is equal to the Circle.

And I alſo ſay, that the Line *G C*, which the Line *B C* cuts off from the Diameter *G A*, is equal to one fourth Part of the Circumference.

Several of theſe Lines are incommenſurable, as *G P*, *G B*, *G S*, but their Squares may be found, which will anſwer the Purpoſe; for if the Diameter is but known, that is all which is neceſſary.

Fig. II.* In this Circle are ſome Additions, but the Triangle *G S A*, is the ſame.

Poſt. About the Centre *G*, with the Diſtance of the Diameter
3 of 1. *G A*, a Circle is deſcribed, which cuts the Secant Line *G S*, in the Point *V*.

31 of 1. And through that Point *V*, a Line is drawn parallel to the Line *S A*, and cuts the Diameter in the Point *R*.

Again,

About the Centre *G*, with the Diſtance of the Line *G R*, a Circle is deſcribed, which cuts the Secant Line *G S*, in the Point *B*.

And through that Point *B*, a Line is drawn parallel to the Line *V R*, and cuts the Diameter in the Point *C*.

Now here are three right angled Triangles, *G B C*, *G V R*, *G S A*, the Angle *A G S*, is common to them all,
Coroll. the Angles *C*, *R*, *A*, at the Diameter, are all right Angles,
32 of 1. conſequently, the remaining Angle in each, muſt be equal to one another.

Def. 1 of 6. Therefore theſe three Triangles are ſimilar and proportional.

4 of 6. And ſince the ſide *S A*, in the Triangle *G S A*, is equal to half the Diameter *G A*, the other two ſides *V R*, *B C*,

in

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[*] I have only drawn the Figure again, to prevent perplexing it, by having too many Lines in one Figure.

in the other two Triangles *G V R*, *G B C*, will alſo be equal to half of their Baſes, *G R*, *G C*.

In the right angled Triangle *G S A*, the ſquare of the ſide *G S*, which is oppoſite the right Angle *G A S*, is equal to the ſquares of the other two ſides *G A*, *A S*. 47 of 1.

And ſince the ſide *S A*, is equal to half the ſide *G A*, the ſquare of the ſide *S A*, will be equal to one fourth 4 of 2. part of the ſquare of the ſide *G A*, conſequently, the ſquare of the ſide *G S*, muſt be equal to five times the ſquare of the ſide *S A*; therefore, if from the ſquare of the ſide *G S*, one fifth part be taken from it, the remainder will be the ſquare of the ſide *G A*, and ſo it will be in the other two triangles *G V R*, *G B C*.

Let the Diameter be - - - - - 60

Then in the Triangle *G V R*, the ſquare of Def. 15 of 1. the ſide *G V*, which is equal to the Diameter *G A*, and is oppoſite the right Angle *G R V*, is - 3600

Take from it one fifth for the ſquare of the ſide *V R* - - - - - - - 720

Gives this for the ſquare of the ſide *G R* - 2880

And ſince the ſide *G R*, is equal to the Line Def. 15 of 1. *G B*, I ſay, this is the Contents of the Circle.

In the Triangle *G B C*, the ſquare of the ſide *G B*, which is oppoſite the right Angle *G C B*, and is equal to the line *G R*, is as above - - 2880

Take from it one fifth for the ſquare of the ſide *B C* - - - - 576

Gives this for the ſquare of the ſide *G C* - 2304

The Root of this ſquares come out, as it always will, if the Diameter is a whole Number, and gives this for the Line *G C* - - - 48

Which, I ſay, is equal to one fourth Part of the Circumference.

Before I apply theſe to prove my Propoſition, I muſt make a few more Obſervations.

2 of 12. Circles are to each other, as the ſquares of their Diameters; and ſince it has been proved, that the ſquare of the Chord *G P,* is equal to half the ſquare of the Diameter *G A,* that Circle which has the Chord *G P,* for it's Diameter, muſt be equal to half of that Circle, whoſe Diameter is the Line *G A*; that is, it will be equal to the Semicircle *G P A,* conſequently, the Semicircle *G M P,* muſt be equal to the Quadrant *L G P.*

Now ſince the Semicircle *G M P,* is equal to the Quadrant *L G P,* the Segment (*a*) is common to both; take away that Segment (*a*) and thoſe three Figures (*c*) (*b*) (*d*) which make up, what I call, the curved Figure *G M P,* remains equal to the Triangle *L G P.*

Again,

Since the Semicircle *G M P,* is equal to the Quadrant *L G P,* and the Triangle *L G P,* is equal to the Triangle *G M P,* take away both thoſe Triangles and the Segment (*a*) remains equal to both thoſe Segments (*c*) and (*d*).

And now I will endeavour to apply what has been done, to prove my Propoſition.

The ſquare of the Diameter *G A*, is - 3600

And I have, I think, undeniably proved, that the ſquare of the Line *G B* is - - 2880

Therefore the ſquare of the Diameter *G A*, exceeds the ſquare of the Line *G B* - - 720

Now then if the ſquare of the Line *G B*, is equal to the Circle, the ſquare of the Diameter *G A*, will exceed the Circle in the ſame proportion; and it is evident, that the ſquare of the Diameter *G A*, only exceeds the Circle by thoſe four Corners which the Circle cuts off from it. Therefore, what I have now to do, is, to prove that thoſe four Corners are equal to - - - 720

For which Purpoſe, I deſire that theſe following Obſervations may be attended to.

I have proved that the Segment (*a*) is equal to thoſe two Segments (*c*) and (*d*).

It is ſelf-evident that the Segment (*a*), together with the Corner (*b*), are equal to the Triangle *G M P*.

I have alſo proved, that thoſe three Figures, (*c*), (*b*), (*d*), which make together, what I call, the curved Figure *G M P*, are equal to the Triangle *L G P*, or the Triangle *G M P*, for they are equal.

And either of thoſe Triangles *L G P*, or *G M P*, is equal to one eighth Part of the ſquare of the Diameter *G A*, and therefore, muſt be 450.

And now I will try what I can make of them.

See my Second Figure.

$$a = c + d$$

$$a + b = 450$$

$$b + c + d = 450$$

$$a + b + c = 450 + c$$

$$a + b + d = 450 + d$$

Therefore - - $b = 450 - c - d$

$$b + c = 450 + c - a$$

$$b + d = 450 + d - a$$

Theſe added are $3b + c + d = 1350 - 2a$

And ſince $a = c + d$

They will be - - $3b + a = 1350 - 2a$

$$\left.\begin{array}{l}\text{And ſince } a + b = 450 \\ \text{So muſt } 3a + 3b = 1350\end{array}\right\} \text{therefore } 2a + a = 1350 - 3b$$

Now if $2a$ be added on each ſide the firſt Equation, and $3b$ on each ſide the ſecond, they will be

$$2a + 3b + a = 1350$$

and $3b + 2a + a = 1350$

I think, it cannot be ſaid, that theſe two Equations are not exactly equal to each other, and I deſire it may be obſerved, that theſe two Equations added together are exactly equal to the mean proportion between the circumſcribing, and the inſcribed ſquares.

From what has been here done, it will appear as evident as any one Axiom in EUCLID, that $3b + a$ is equal to $3a$, conſequently that $2a$ is equal to $3b$.

For if we add theſe three Equations together - - -

$$\left\{\begin{array}{l} b + c + d = 450 \\ a + b + c = 450 + c \\ a + b + d = 450 + d \end{array}\right.$$

Since $a = c + d$

We ſhall have - - - $3b + 4a = 1350 + a$

Now, if we take away $3a$ from each ſide that Equation, we ſhall have this $3b + a = 1350 - 2a$

And if we take away $3b + a$, from the ſame we ſhall have this - $2a + a = 1350 - 3b$

And theſe two Equations are the ſame as above, which are exactly equal to each other.

Therefore $3b + a$ muſt be equal to $3a$, conſequently $2a = 3b$; and if we add $2a$ on each ſide, the firſt of thoſe Equations, and $3b$ on each ſide the ſecond, they will ſtand thus: - - - Axiom. 3 of 1.

$$2a + 3b + a = 1350$$
$$3b + 2a + a = 1350$$

The ſame as above, exactly equal to each other, and they divide the mean Proportion between the circumſcribing and the inſcribed ſquares exactly into two equal Parts.

And ſince $2a = 3b$, either ten times (a), or fifteen times (b) muſt be equal to - - - 2700

Conſequently the Segment (a) muſt be - 270

And the Corner (b) muſt be - - - 180

And together they give either of thoſe Triangles *LGP*, or *GMP* - - - - 450

Though this Proof gives me all I want, and is, I think, inconteſtible; yet I will not reſt it upon this Proof only, but I will try to make it out ſome other way.

See my Figures.

Suppoſe the Circle to be equal to - x

The Circle contains four Segments and four Triangles, and each Triangle contains one Segment and one Corner. Therefore - - $x = 8a + 4b$

The Circle and the four Corners which the Circle cuts off from the ſquare of the Diameter *G A*, are together equal to 3600. Therefore - $x + 4b = 3600$

And $x = 3600 - 4b$

Now - $x = 8a + 4b$

And - $x = 3600 - 4b$

Therefore - $2x = 8a + 3600$

And ſince $x + 4b = 3600$, ſo will $3x + 12b = 10800$

Therefore - - $5x + 12b = 8a + 14400$

From this it will, I think, plainly appear, that $12b = 8a$, conſequently $2a = 3b$; for if we take $8a$ from the one ſide of that Equation, and either $8a$ or $12b$ from the other, we ſhall find that the remainders will be exactly the ſame.

Firſt, If we take $8a$ from the one ſide, and $12b$ from the other, we ſhall have remaining - $5x = 14400$

Now ſince - $x = 8a + 4b$

So will - - $5x = 40a + 20b$

Therefore if - - - $40a + 20b = 14400$
Since $a + b = 450$, ſo will - $20a + 20b = 9000$

Which taken from it leaves - $20a = 5400$

That is - $a = 270$
Conſequently $b = 180$

For they muſt make together $a + b = 450$

Now if - $40a + 20b = 14400$
So muſt $b = 180$
Therefore - - $20b = 3600$

Together - - $40a + 40b = 18000$

And ſince $a + b = 450$, ſo is $40a + 40b = 18000$, exactly the ſame; from which, I think, it is evident, that $40a + 20b$, or $5x = 14400$.

Now if, inſtead of $12b$ from the one ſide, we take $8a$ from both, we ſhall find that the remainders will ſtill be the ſame.

For ſince $5x = 40a + 20b$
We ſhall have - $40a + 20b + 12b = 8a + 14400$
Now if we take $8a$ from each ſide of that Equation, we ſhall have - - - - - $32a + 32b = 14400$
And ſince $a + b = 450$, ſo is $32a + 32b = 14400$

From theſe we ſee that $5x$ or $40a+20b=14400$
And alſo that - $4x+16b$ or $32a+32b=14400$
Therefore $16b=x$, and ſince $x=8a+4b$
So will - - $16b=8a+4b$
Now if we take $4b$ from
each ſide, we ſhall have $12b=8a$
Conſequently - $3b=2a$

Again,
Since - - - $32a+32b=14400$

And ſince $16b=x$
We ſhall have - $32b = 2x$
Which is ~~the~~ two fifths of the above - - 5760
Conſequently - - $b=180$
And we ſhall have - $32a = 3x$
Which is three fiſths of the ſame - - 8640
Conſequently - $a=270$

Therefore $12b=2160$, and $8a=2160$ and $5x=14400$

Conſequently $3b=540$, and $2a=540$, and $x=2880$
Again,
Since $8a+4b=x$, and ſince $16b=x$
$8a+12b$ will be equal to x, and half of x

$8a=2160$		$x=2880$
$12b=2160$	half of	$x=1440$
4320	exactly equal	4320

Surely after all this, no one can doubt the truth of it; however, to remove every ſhadow of a doubt, I will endeavour to prove it another way.

See my Figures.

$$\text{Since } a+b=450 \left\{\begin{array}{l} 8a+8b=3600 \text{ The circumſcribing ſquare.} \\ 6a+6b=2700 \text{ The mean Proportion.} \\ 4a+4b=1800 \text{ The inſcribed Square.} \end{array}\right.$$

And $8a+4b$ is equal to the Circle.

If the Circle divided the Triangle GMP into two equal Parts, the mean Proportion $6a+6b$, or 2700, would be exactly equal to the Circle; for then either (a) or (b) would be equal to 225.

But the Circle does not divide the Triangle GMP, in that manner; and whatever the Circle takes more than the mean Proportion 225 from the Triangle GMP, juſt ſo much as it takes more than that from (b), it gives to (a).

The ſquare of the Diameter, or the whole Figure contains - - - - - - $8a+8b$

One half of which - - - - - $4a+4b$

And one fourth Part of the ſame - - $2a+2b$

Thoſe added together give this - - $6a+6b$ for three fourths of the whole Figure; and is exactly the mean Proportion between the two Squares.

$6a + 6b$

So far, we may ſuppoſe the Corners and Segments to be equal to each other, or any thing, for together they will make the ſame.

But now we come to the Circle, they will appear what they really are.

For if we add this $2a - 2b$ to the mean Proportion - - - - $2a - 2b$

We ſhall have the Circle - - - - $8a + 4b$

Here we ſee, the Circle no ſooner makes its appearance, than a quantity equal to $2b$, is taken away from the mean Proportion of the one, which muſt be given to the other, for here cannot any thing be loſt; and ſince the Circle contains thoſe eight Segments, we ſee the whole of it is given to the Circle, for what remain now to complete the whole Figure, are four real, ſeparate, diſtinct, and unconnected quantities, which now appear for themſelves - - $4b$

And which added to the Circle, complete the whole Figure.

And they are $8a + 8b$ ſtill - - - $8a + 8b$

From this it is very evident, that thoſe eight Segments have taken away from thoſe eight Triangles, a quantity equal to $2b$, more than the mean Proportion, and that quantity is exactly equal to one fourth Part of $8b$.

From

Therefore, if we divide the mean Proportion between (a) and (b) by 5, and take the Product from the mean Proportion, we ſhall have the Corner (b), and if we add that Product to the mean Proportion, we ſhall have the Segment (a).

$a + b$ together are - - - - -	450
The mean Proportion between them -	225
Divided by 5 gives this - - -	45
Which taken from the mean Proportion, gives this for the Corner (b) - -	180
The mean Proportion between (a) and (b) -	225
And this added to it - - -	45
Gives this for the Segment (a) - - -	270

Exactly what I have all along made them both, and ſufficiently confirms my Propoſition.

For from theſe Proofs, it is, I think, ſufficiently evident, that thoſe four Corners which the Circle cuts off from the ſquare of the Diameter $G A$, are equal to 720, for four times the Corner (b) 180, give for thoſe four Corners 720.

Conſequently the Circle muſt be equal to the ſquare of the Line $G B$ - - - - -	2880

It is alſo evident, that the inſcribed ſquare within the Circle (which is equal to half the ſquare of the Diameter) together with the four Segments, muſt give the Contents of the Circle.

Half the ſquare of the Diameter, or the inſcribed ſquare is — — — — — 1800

Four times 270, which has been proved equal to the Segment (*a*), give this — — 1080

And together they give this - - 2880 for the Contents of the Circle.

And is exactly equal to the ſquare of the Line *G B*.

Again,

Since the ſquare of the Diameter *G A* is - 3600

And ſince the ſquare of the Line *G B* is - 2880

The ſquare of the Diameter *G A*, exceeds the ſquare of the Line *G B* - - - - 720

Which is exactly one fourth Part of the ſquare of the Line *G B*.

Now then, if the ſquare of the Line *G B* is equal to the Circle, thoſe four Corners which the Circle cuts off from the ſquare of the Diameter *G A*, muſt be equal to one fourth Part of the Circle: Therefore, if one fourth Part of the Circle be taken away from the ſquare of the Diameter *G A*, which let be the Quadrant *I L A*, the remainder will be equal to the Circle, that is, thoſe three ſquares *LPSA*, *GMPL*, *KGLI*, with the Corner *N* 180 (for the Corner (*b*) has been proved equal to 180) will give the Contents of the Circle.

The ſquare *LPSA*, is one fourth Part of the ſquare of the Diameter *GA*, and therefore is -	900
The ſquare *GMPL* is the ſame - -	900
The ſquare *KGLI* is the ſame - -	900
The Corner *N* - - - -	180
And they give this for the Contents of the Circle	2880
And the Quadrant *ILA*, one fourth Part of the Circle, is exactly one fourth Part of the ſquare of the Line *GB* - - - - -	720
And completes the ſquare of the Diameter *GA*	3600

Therefore thoſe four Corners are equal to one fourth Part of the Circle.

Nothing more, I think, is now neceſſary to be done, but to prove, that the Line *GC*, is equal to one fourth Part of the Circumference.

Every one knows, who knows any thing about it, that the Circumference of a Circle multiplied by one fourth Part of the Diameter, muſt give the Contents of the Circle.

I have proved that the Line *GC* is - -	48
Four times which for the Circumference -	192
Multiplied by one fourth Part of the Diameter	15
Give this for the Contents of the Circle -	2880

Which ſufficiently proves, that the Line *G C* is equal to one fourth Part of the Circumference.

I think that enough has now been ſaid and done, to prove the truth of my Propoſition, that the ſquare of the Line *G B* is equal to the Circle; and alſo, that the Line *G C* is equal to one fourth Part of the Circumference.

For I truſt, that it will be found impoſſible, that every Part ſhould agree in this extraordinary Manner, and not be true.

I ſhall now only add a few Obſervations and Inferences which may be drawn from it.

Firſt,

I will juſt obſerve, that one tenth Part of the ſquare of the Diameter, or of the whole Figure 3600 is — — — — — 360

One tenth Part of the mean Proportion between the circumſcribing and inſcribed ſquares 2700, is the Segment (*a*) — — — — — 270

And one tenth Part of the inſcribed ſquare 1800, is the Corner (*b*) — — — — 180

And their common Ratio is 90. ——

One tenth Part of the ſquare of the Diameter is 360

Two tenths give the four Corners — — 720

Three tenths give the four Segments — 1080

Four tenths give half the Circle — — 1440

Which added together give the ſquare of the Diameter — — — - 3600 ——

And their common Ratio is 360. ——

Second,

That the fourth Part of the Circumference, or the Line *G C,* is in the ſame exact Proportion with the Diameter *G A*; that the Circle, or the ſquare of the Line *G B,* is with the ſquare of the Diameter *G A,* for they are exactly four fifths, the one of the Diameter, and the other of its ſquare.

For four fifths of the Diameter 60 are — 48

Which is the Line *G C,* or one fourth Part of the Circumference.

And four fifths of the ſquare of the Diameter 3600 are . — — — — — 2880

Which is the ſquare of the Line *G B,* or the Contents of the Circle.

Third,

Though the Proportion exceeds by much any I have ever ſeen, yet I believe that I may venture to aſſert, that the true Proportion between the Diameter and the Circumference, is as one is to three two-tenths, ſo is the Diameter to the Circumference.

For as 1 : 3, 2 : : 60 : 192,

Which is exactly four times the Line *G C,* 48.

FINIS.

APPENDIX.

SINCE I finiſhed my Solution, I have been conſidering what could be the Reaſon that this Problem has been ſo long undiſcovered, and I think it has been owing to ſomething of the following Nature.

Mathematicians divided the Circumference of the Circle into 360 equal Parts or Degrees, and they have ſuppoſed that there are as many Angles at the centre of the Circle, as there are Diviſions upon it's Limb, which I think will be found not true. Afterwards they gave us Tables of Logarithms, the one for Numbers, which I believe may be right, the other of Sines and Tangents for the Quadrant, even ſo low as to one hundred Part of a Degree, in which I think there is not any one Number right, except that for the Radius, they have as before taken it for granted, that there are Angles when no ſuch Angles really exiſt, and they have divided the Quadrant accordingly; by which means thoſe Numbers, for very acute Angles are too little, and when we get paſt thoſe acute Angles they are then too great, till we come to an Angle of 45 Degrees, and their Complements are accordingly.

This I take to be the Caſe, and here I think will be found the Error.

See my firſt Figure.

If from the Point *B*, we cut off 45 Degrees from the Arch *B A*, neither the remaining Arch or Tangent are, I think, any meaſure for any Angle in this Circle; and every Degree more which we take from this remaining Arch and Tangent towards the Diameter, the remaining Tangents will every one of them be Tangents of different Circles; for as the Secant Line approaches nearer to the Diameter, it keeps uniting more and more with it, till they are wholly united together; and whenever a Tangent Line is ſo far reduced, that its Secant Line unites with the Radius before it reaches the centre of the Circle, it can not then be the Meaſure of any Angle in that Circle, but muſt be a Tangent to a leſſer Circle, conſequently to a greater Angle than is laid down for it by thoſe Tables; and I think it will be found, that a Secant Line drawn from this remaining Tangent, will unite with the Radius, before it arrives at the centre of this Circle; and ſo ſoon as ever it begins to unite with the Radius, it muſt there, I think, determine the centre of a Circle for that Tangent. If this is the caſe, the method which has been taken to come at the Circle, has had the quite contrary effect, and has driven them the farther from it; for the more they increaſed the Number of ſides for a Polygon to come nearer the Circle, the worſe they were; for as the Tangent muſt make half of one of thoſe ſides, the leſs the Tangent, if taken from thoſe Tables, the greater the error.

It is very true that we may ſuppoſe ſuch Angles, and we may do ſome things very well upon ſuch a ſuppoſition; for where we have two fixed Points as the centre of a Quadrant and its Limb, they will very truly direct us to a third; but it does not follow, that becauſe we can count the Degrees upon it's Limb, that there is that Angle at the centre. Whether this is really ſo, or whether there may be ſome other cauſe why they failed, I will not take upon me to determine; and I wiſh it to be underſtood that I am not now attempting to prove any thing, I think, that I have proved enough for the preſent; for if my Solution is true, which I think cannot be contradicted, it is a ſufficient Proof that thoſe Tables are ſomehow wrong, for they give Tangents too little for very acute Angles. However, this will pretty nearly ſettle the matter.

If thoſe Tables will give a Tangent of 4, 8 for an Angle, if there is ſuch a one; or, however, for an Arch of 8 Degrees 24 Minutes upon this Circumference, we ſhall perfectly agree; if not, I think that I may ſafely ſay that they are wrong; but whether as I have above ſuppoſed, I ſhall leave for others to determine.

I will turn once more to the Arch *B A*, as there is ſomething very particular in it, which I cannot paſs by unnoticed.

See my firſt Figure.

I muſt firſt obſerve, that I now ſuppoſe the Diameter *G A* to be — — — — — — 120

Then according to my Solution, the Circumference will be 384

For as 1 : 3, 2 : : 120 : 384,

Conſequently one fourth Part of the Circumference, or the Arch *P A* will be — — — — 96

And the Line *G C* will be the ſame.

If from the Point *B* we cut off 45 Degrees from the Arch *B A*, and there draw a Secant Line, we ſhall want nothing more than the remaining Arch and Tangent to come at the Circle.

This remaining Tangent will be found to be exactly equal to one tenth Part of the Line *G C*, or the Arch *P A*, that is, to 9, 6

And this remaining Arch will direct us to a Chord to an Arch of 9 Degrees.

The Arch *B A* will be found to be 53 Degrees 24 Minutes; if we change theſe Minutes for Decimals, then this Arch *B A* will be — — — — — — 53, 4
And if we take from it — — — — 45

We ſhall have this — — — — — 8, 4
for the remaining Arch.

Now, if the remaining Tangent is equal to an Arch of 9 Degrees, an Arch of 9 Degrees will meaſure upon a right Line 9, 6.

And if an Arch of 9 Degrees meaſures 9, 6 upon a right Line, ſo will an Arch of 8, 4, meaſure 8, 96 upon a right Line.

For as 9 : 0, 6 : : 8, 4 : 0, 56.

An Arch of — — — — 9
Exceeds an Arch of — — — 8, 4

By an Arch of — — — — 0, 6

But a right Line of — — — — 9, 6
Exceeds a right Line of — — — — 8, 96

By a right Line of — — — — 0, 64

Therefore an Arch of 0, 6 will meaſure 0, 64 upon a right Line, and whether it does ſo or not, may eaſily be known.

For as 0, 6 : 0, 64 : : 9 : 9, 6 the Arch of 9 Degrees.
Or as 0, 6 : 0, 64 : : 90 : 96 the Arch of 90 Degrees.
Or as 0, 6 : 0, 64 : : 360 : 384 the Circumference.

I will now try to come at the Chord to an Arch of 9 Degrees; but ſince this Chord is a Surd, as I believe all other Chords to whole Numbers upon the Arch are, except the Diameter, I ſhall not here be perfectly exact.

An Arch of	9, 0
Exceeds an Arch of	8, 4
By an Arch of	0, 6
But a right Line of	9, 6
Cannot exceed a right Line of	8, 4
By an Arch, but by a right Line of	1, 2

Therefore, if we can but come at the Difference that there is between an Arch of 1, 2, and a right Line of 1, 2, that Difference added to 8, 4, will give us the Chord to an Arch of 9 Degrees. And which I will now endeavour to come at as near as I can.

An Arch of	9,
Exceeds an Arch of	8, 4
By an Arch of	0, 6

And an Arch of 0,6 is equal to a right Line of 0, 64.
Therefore as 9 : 0, 64 : : 1, 2 : 0, 0853, &c.

Now if to	8, 4
We add this	0, 0853
We ſhall have this	8, 4853

for a Chord to an Arch of 9 Degrees.

And if from - - - - - - 9,6
We take this - - - - - - 8, 4853

We ſhall have this - - - 1, 1147
for what an Arch of 9 Degrees, exceeds a Chord to an Arch of 9 Degrees.

And now we will ſee how the whole agrees.

The remaining Tangent - - - - 9, 6
Multiplied by - - - - - - 50,
Will give us this - - - - - - 480,
Which is the Sum of the four Sides of this circumſcribing Square.

The remaining Tangent - - - - - 9, 6
Multiplied by - - - - - - - 40,
Will give us this - - - - - - 384,
Which is the Circumference of this Circle; and ſince they have both one common Multiplier 30, the Circle muſt be one fifth Part leſs than the circumſcribing Square.

The Chord of 9 Degrees - - - - 8,4853
Multiplied by - - - - - - 10,
Will give us the Chord *PA* - - - - 84, 853
And ten times 1, 1147, which is the Difference between the Arch and Chord of 9 Degrees, added to it - 11, 147
Will give us this for the Arch *PA* - - 96,

Now, whether this is not near enough to ſatisfy any one, that this remaining Arch and Tangent cannot be otherwiſe, may eaſily be known, as the Chord *PA* is a Root of this Square - - - - - - - 7200

And if we ſuppoſe the Diameter to be - - 60
One half of the above Chord will be the Chord *PA* - 42,4265
And one half of the above Difference between the Arch and Chord added to it - - - 5·5735
Will give us this for the Arch *PA* - - 48,

Since this Chord is always a Surd, it cannot be come at exactly this way: But, however, though they are Surds, they agree within about ten thouſand Part of an Unit, and which may eaſily be known, as this Chord is the Root of this Square - - - - 1800

I had a mind juſt to mention theſe Things, as they may be Leadings to ſomething more; and if my Solution ſhould be received ſo as to give me any Encouragement, I may, perhaps, at ſome future Day purſue them further, and be able to prove what I have here advanced; but my Attention has for ſo long a Time been ſo much taken up with this Problem, that for the preſent it is abſolutely neceſſary for me that I give it up.

Therefore I will now conclude with only propoſing one Proof more for my Solution, and which I think is worth them all, though I know it will not be admitted as ſuch, neither do I think my Solution wants it. But if any one ſhould think it worth his Trouble to make the Experiment, he will find that my Solution will entirely correſpond in Practice. And why that ſhould not be admitted as Proof, I cannot conceive, for I think that Theory and Practice ſhould go Hand in Hand; that there is little Uſe in the one without the other, and that only ſo far as they both agree, juſt ſo far, but no farther, the ARTS and SCIENCES are perfect.

FINIS.